Film-Cooling Effectiveness and Skin Friction in Hypersonic Turbulent Flow

AUBREY M. CARY JR.* AND JERRY N. HEFNER†
NASA Langley Research Center, Hampton, Va

Experimental equilibrium temperatures and skin friction at the surface of a flat plate cooled by two-dimensional, tangential slot injection are presented for a Mach 6 mainstream. Effects of slot height, slot mass-flow rate, injection gas temperature, and heat conduction to the slot are investigated. Experimental skin friction and effectiveness data in general agree well with predictions from a finite-difference method. Film cooling at Mach 6 was found significantly more effective than at lower speeds, with large reductions in friction drag measured downstream of the slot. These results indicate the necessity for a new evaluation of film-cooling systems for hypersonic vehicles.

Nomenclature

C_f = skin-friction coefficient, τ_w/q_∞
M = Mach number
P = pressure
q = dynamic pressure, $\frac{1}{2}\rho u^2$
Re = unit Reynolds number, $\rho_\infty u_\infty/\mu_\infty$
S = slot height
T = temperature
$T_{j,AV}$ = average slot total temperature, $\frac{1}{S}\int_0^S T_{t,j} dY$
t = slot lip thickness
u = velocity
x = distance downstream of slot
Y = distance above plate surface
Y' = distance above top of slot lip
δ = boundary-layer velocity thickness (at $u/u_\infty = 0.995$)
δ^* = displacement thickness
ε = effectiveness parameter, $(T_{t,\infty} - T_{eq})/(T_{t,\infty} - T_{t,j})$
θ = momentum thickness
λ = mass flow parameter, $\rho_j u_j/\rho_\infty u_\infty$
μ = dynamic viscosity
ρ = density

Subscripts

cl = cooling length
eq = equilibrium
j = slot
o = without slot
t = total or stagnation
w = wall
∞ = freestream

Introduction

FILM cooling is one means of decreasing the operational surface temperature of a high-speed flight vehicle below the radiation equilibrium temperature. With the surface temperature at a lower level, lighter and less expensive materials can be used in structural fabrication. Transpiration and convective cooling systems have been evaluated for flight application,[1] but a paucity of experimental information and analytical prediction methods prevents such an evaluation for film-cooling systems. Results from film-cooling experiments for subsonic mainstream flow (see Refs. 2 and 3 for a summary of low-speed results) and from one experiment[4] at Mach 3 have been used[5,6] to estimate the efficiency of film cooling in high-speed flow, and it was found that film cooling was inferior to transpiration cooling. However, recent investigations[3,7-9] at Mach 6 show significantly more effectiveness for film cooling in high-speed turbulent flow than at lower speeds or than indicated by extrapolations of low-speed results to high speeds.[6] While effectiveness data presented in Refs. 3, 7, and 8 are questionable,[2] they suggest increased efficiency for film-cooling systems for high-speed aircraft.

The present paper reports on accurate measurements at Mach 6 of surface equilibrium temperature (and thus effectiveness) and skin friction downstream of a two-dimensional sonic slot with tangential air injection into a thick, turbulent boundary layer. The effects on skin friction and effectiveness of slot height, slot mass-flow rate, injection gas temperature, and heat conduction in the slot are presented for a significant range of each variable. The present data are compared with predictions from a finite-difference method[2] which utilizes eddy diffusivity and mixing length concepts to solve the compressible turbulent boundary-layer equations with tangential injection. The present results are also compared with previous experimental data and correlations.

Test Apparatus and Model

A fiberglass flat plate (35.5 cm wide and 91.4 cm long) with end plates (see Fig. 1) was mounted parallel with an recessed below the flat tunnel wall of the Langley 20-in. Mach 6 wind tunnel.[10] Flush-mounted thermocouples along the center line determined plate surface temperatures and three floating element gages located 6.36 cm off centerline determined skin friction. Pressure orifices were located 3.81 cm off centerline at four x-locations. The slot flow ejected tangentially along the surface of the flat

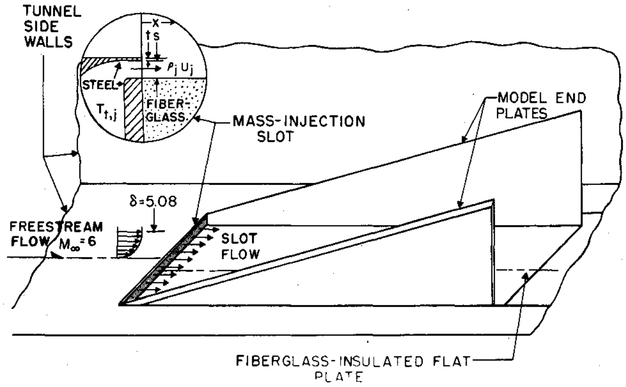

Fig. 1 Two-dimensional film-cooling model and slot configuration.

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* Aerospace Engineer, Gas Dynamics Section, Hypersonic Vehicles Division. Member AIAA.

† Aerospace Engineer, Configuration Flow Fields Section, Hypersonic Vehicles Division. Member AIAA.

plate and was uniform over a midspan of at least 20 cm. The slot configuration, shown in the insert of Fig. 1, could be adjusted to provide slot heights (S) of 0.159, 0.476, and 1.11 cm while the lip thickness (t) was fixed at 0.159 cm. The slot mass-flow rate ($\rho_j u_j$) was regulated so that the ratio of measured slot mass-flow rate to calculated freestream mass-flow rate (λ) ranges from 0.03 to 1.60. The freestream total temperature ($T_{t,\infty}$) and unit Reynolds number per cm (Re/cm) were 478° K and 0.24×10^6, respectively. The temperature of the slot flow was controlled by using a liquid nitrogen heat exchanger and varied from 166° K to 311° K.

Measurements and Test Procedures

Surface and pitot pressures were measured by multirange capacitance-type pressure transducers calibrated to better than 1% accuracy. The tip of the pitot probe was 0.025 cm high and 0.092 cm wide; the total temperature probe was a shielded thermocouple with base bleed and a circular tip of 0.152 cm diam. The temperature probe was calibrated for Mach numbers from 2 to 8.5 and Reynolds numbers per cm from 0.016×10^6 to 0.4×10^6, and no temperature correction was required over this range of conditions. Velocities were calculated by using the probe values of temperature and pressure along with the assumption of constant static pressure across the slot exit and tunnel wall boundary layer.

A plate surface temperature was considered to be in equilibrium when it changed less than 0.1% over a time interval of 100 sec. Calculated conduction error in the measured equilibrium temperature was less than 1% for the most severe case and was thus neglected. Generally, only the forward portion of the plate surface ($x < 46$ cm) reached equilibrium temperature in the available test time (≈ 1000 sec).

The null-type skin friction balances (Kistler) had a floating element diameter of 0.94 cm, a surrounding gap width of 0.01 cm, and multiple sensitivities of 0.15 g/cm^2, 1.5 g/cm^2, and 15 g/cm^2. The balances were statically calibrated before and after the test program by applying a known load and measuring the voltage necessary to null the balance. The calibration remained constant throughout the tests within 3%. Water-cooled jackets maintained the balance internal temperature below 360° K and thus minimized errors due to temperature sensitivity.

Results and Discussion

Boundary-Layer and Slot Profiles

The tunnel wall boundary-layer velocity and total temperature profiles at the slot location for the present freestream conditions are shown in Fig. 2. The momentum thickness Reynolds number is approximately 4.4×10^4, and the velocity thickness (δ) is approximately 5.08 cm. The temperature profile is similar to

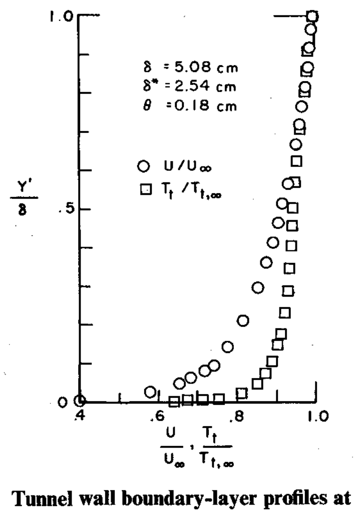

Fig. 2 Tunnel wall boundary-layer profiles at the slot lip.

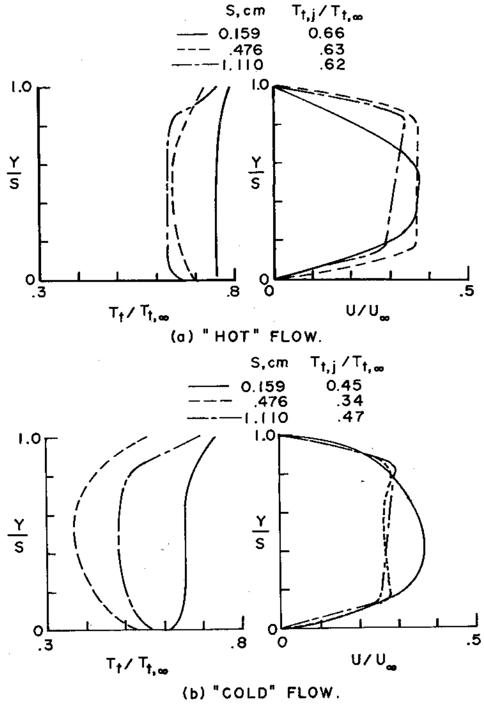

Fig. 3 Measured slot profiles; $\lambda \approx 0.05$ for "hot" flow, $\lambda \approx 0.06$ for "cold" flow.

other turbulent profiles measured on nozzle walls[11] and would show a quadratic relation with velocity in terms of the Crocco variables $[(T_t - T_w)/(T_{t,\infty} - T_w)$ against $u/u_\infty]$.

Examples of velocity and total temperature profiles measured at the slot exit for both room-temperature air injection ("hot" flow) and cooled air injection ("cold" flow) are presented in Fig. 3 for each of the three slot heights. For these data, the slot flow rate was regulated such that the static pressure at $Y/S = 0$ was the same as the freestream static pressure. Values of $T_{t,j}/T_{t,\infty}$ given on the figure were determined from measured values of temperature in the slot manifold (see Fig. 1) and freestream stagnation temperature; deviations in the slot temperature profiles from $T_{t,j}/T_{t,\infty}$ result mainly from conduction of heat from the mainstream flow through the slot lips and into the slot flow. This conduction effect is most pronounced in the "cold" flow profiles and in particular for the smallest slots ($S = 0.159$ cm). Of the velocity profiles shown in Fig. 3, only the $S = 0.159$ cm slot profile has almost fully developed channel flow. These slot profiles along with the tunnel wall profiles are used as inputs for finite-difference calculations of the downstream flow presented later.

Surface Pressure Distribution

Typical surface static pressures shown in Fig. 4 were measured at four locations downstream of the slot exit for various λ and showed that mass injection does not significantly affect the surface pressure except in the vicinity of the slot. Near the slot ($x = 0.635$ cm) the injected mass strongly influences the pressure, and the pressure level corresponds to the degree of expansion of the flow

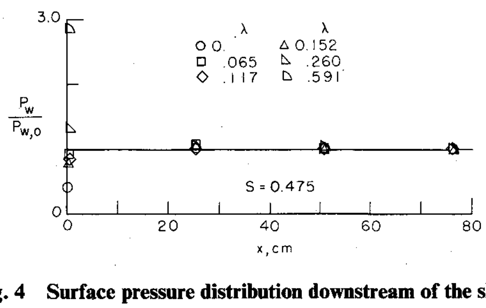

Fig. 4 Surface pressure distribution downstream of the slot.

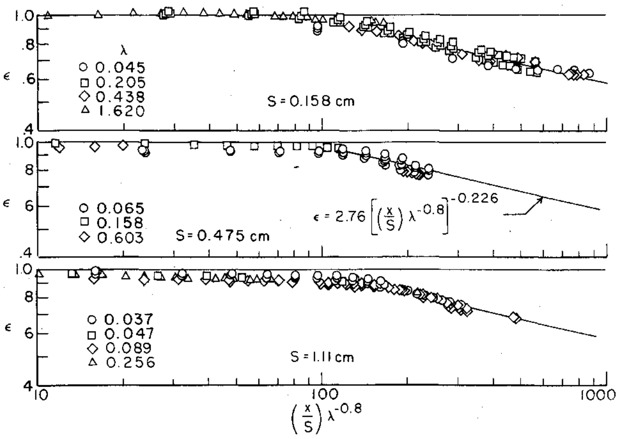

Fig. 5 Film-cooling effectiveness; $T_{t,j}/T_{t,\infty} = 0.63$.

from the slot. The surface pressure approximately equals the stream static pressure ($P_w/P_{w,o} \approx 1$) for a small range of λ and is hereafter referred to as "matched" pressure conditions.

Effectiveness

A summary of the measured surface equilibrium temperatures for $T_{t,j}/T_{t,\infty} = 0.63$ is presented in Fig. 5 in a form which correlated similar data in Refs. 3, and 7–9. The measured slot manifold (stagnation) temperature was used to reduce surface equilibrium temperature to effectiveness, even though, as shown in Fig. 3, the temperature is not, in general, constant across the slot. Data are presented for the three slot heights with constant lip thickness ($t = 0.16$ cm) and various values of injection flow rate. The effectiveness data correlate in a relatively narrow band for the slot mass-flow rates (all sonic injection) and slot heights of this investigation. The best power-law fairing of the data for all three slot heights and $(x/S)\lambda^{-0.8} > 100$ is included in Fig. 5. Thus the present range of slot height and mass-flow rate do not significantly influence the correlation of effectiveness for $(x/S)\lambda^{-0.8} > 100$. Effectiveness apparently decreases with increasing slot height for $(x/S)\lambda^{-0.8} < 100$ (see Fig. 5). However, this effect is thought to be predominantly caused by heat conduction from the freestream flow through the metal slot lip and into the slot flow rather than an effect of slot height.

Cooling the slot flow alters the correlation from that shown in Fig. 5. Effectiveness data for $T_{t,j}/T_{t,\infty} = 0.43$, correlated as in Fig. 5, are shown in Fig. 6. Data are presented for each slot height at the "matched" pressure condition and at a higher value of λ for the largest slot. The data again correlate for all three slots and the λ range, but the effectiveness is lower and decreases more rapidly than for the higher injection temperature of Fig. 5. The equation for the power-law fairing of the data is shown in Fig. 6. This decrease in efficiency is partially due to heat conduction through the slot lips in that the temperature in the slot is higher than the slot manifold temperature (see Fig. 3); however, when an average slot total temperature ($T_{j,AV}$, see nomenclature) is used to define effectiveness, the slot cooling efficiency is still significantly less than at the higher slot-to-freestream total temperature ratio. Therefore, empirical effectiveness correlations, such as presented herein and in Refs. 3, and 7–9, should depend on $T_{t,j}/T_{t,\infty}$. Since all of these data were for Mach 6 mainstream

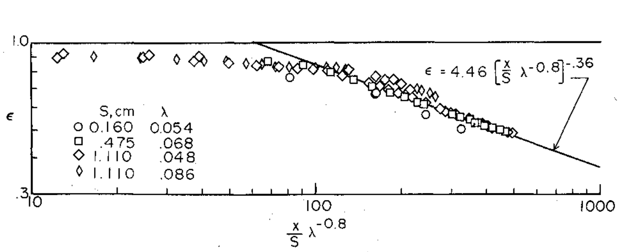

Fig. 6 Film-cooling effectiveness; $T_{t,j}/T_{t,\infty} = 0.43$.

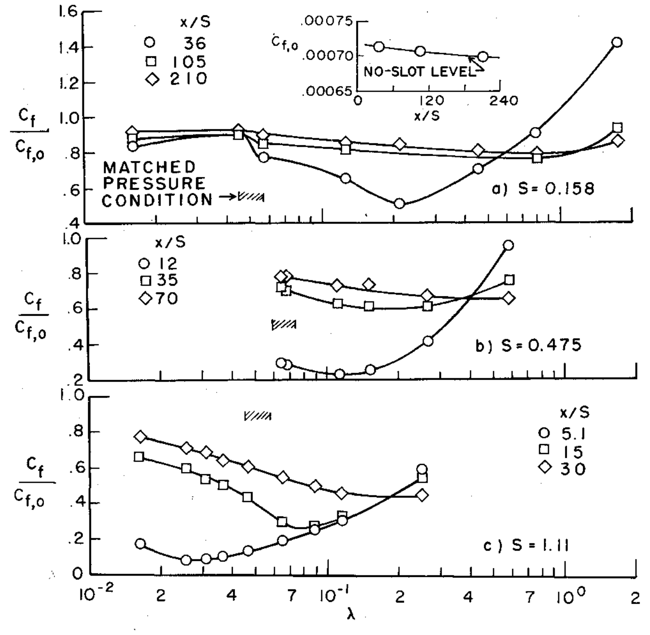

Fig. 7 Skin friction downstream of the slot; $T_{t,j}/T_{t,\infty} = 0.63$.

flow, the correlation may also depend on Mach number. Heat conduction from the freestream to the slot flow is probably present in flight and will complicate predictions of slot effectiveness.

Skin Friction

Local surface skin-friction coefficients with the plate surface temperature at equilibrium and $T_{t,j}/T_{t,\infty} = 0.63$ are shown in Fig. 7 as $C_f/C_{f,o}$ against λ where $C_{f,o}$ is the value of the local skin-friction coefficient without the slot present (see insert of Fig. 7a). Slot mass injection generally reduces the level of skin friction downstream of the slot with the greatest reduction near the slot exit. Increasing the slot mass flow rate first decreases the local skin friction to a minimum; thereafter, the skin friction monotonically increases. The minimum skin friction does not in general occur at the "matched" pressure condition but at a higher slot flow rate. Skin-friction data could not be simply correlated to account for variation in λ and slot height as was true for effectiveness. However, it is obvious from Fig. 7 that significant reductions in skin friction downstream of the slot are possible.

Finite-Difference Method Predictions

The present effectiveness and skin-friction data can be compared with predictions from an implicit finite-difference method[2] when initial slot and boundary-layer profiles are available for nearly "matched" pressure conditions. In Ref. 2 the partial-differential equations for the mean motion of a compressible turbulent boundary layer with tangential injection are solved by an implicit finite-difference procedure with the following physical assumptions: 1) air-to-air injection, 2) constant pressure field across the boundary layer ($dP/dY = 0$ everywhere, no shocks), and 3) thin slot lip (no flow separation or recirculation). The constant pressure field assumption is approximately satisfied for the present data at the "matched" pressure condition for which experimental slot profiles are available (Fig. 3). Predictions from the finite-difference method for this "matched" pressure condition are compared with experimental effectiveness data in Fig. 8 and corresponding skin-friction data in Fig. 9 for each slot height and two slot-to-freestream total temperature ratios.

Although the thin-lip assumption of the theory is violated to some extent for all slots, predictions of the effectiveness shown in Fig. 8 show good agreement with data through the entire range of $(x/S)\lambda^{-0.8}$ at both temperature ratios except for the smallest slot ($S = 0.159$ cm). Here, the lip thickness and slot height are

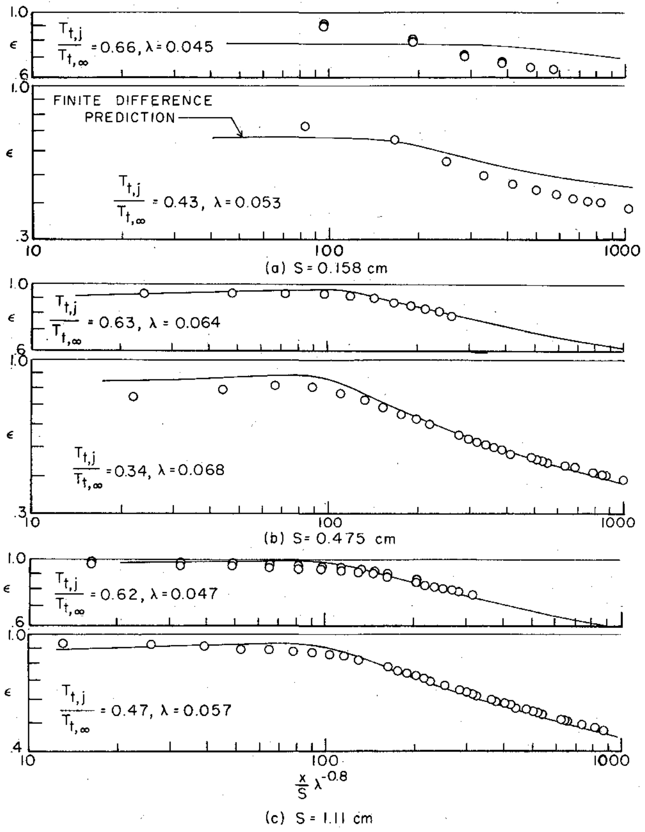

Fig. 8 Comparison of effectiveness data with finite-difference method predictions.

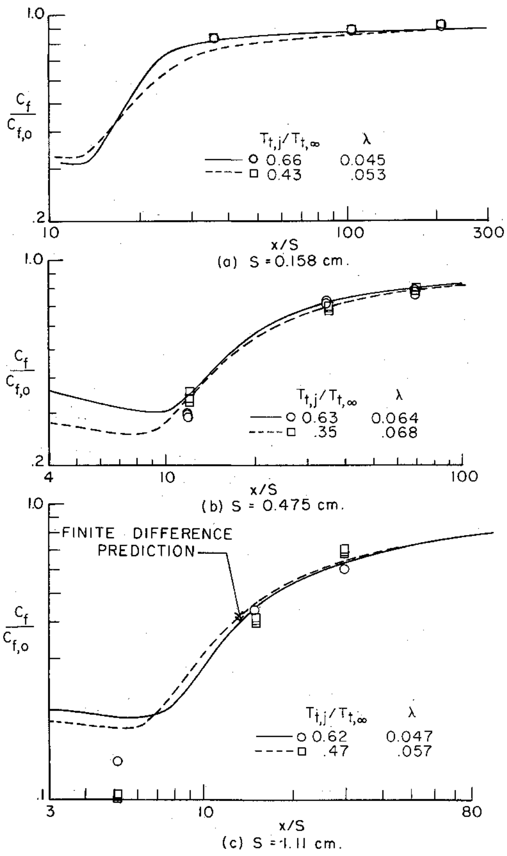

Fig. 9 Comparison of skin friction with finite-difference method predictions.

equal; the recirculation region behind the slot lip is nearer the surface and may increase mixing and thus decrease effectiveness below the predicted level. Also, slot profiles are least accurate for the 0.159-cm slot because of the small slot size relative to probe dimensions. The good agreement between the finite-difference prediction and effectiveness data for the "matched" pressure condition suggests that good predictions of effectiveness for other than "matched" pressure conditions are possible since all the present effectiveness data correlate as ε against $(x/S)\lambda^{-0.8}$.

Predictions of the corresponding skin-friction data shown in Fig. 9 again agree well with the data even for the smaller slot. The one exception in the present data is the $S = 1.11$ cm slot (Fig. 9c) at about five slot heights downstream; here the theory substantially overpredicts the experimental skin friction. This disagreement may be caused by a small separated region on the plate (indicated by slot profiles near the lower slot lip) resulting in lower measured skin friction. Comparing skin-friction data for the two different slot flow temperatures with the same slot height in Fig. 9 indicates that the skin-friction coefficient is little affected by slot flow temperature. The accuracy of predictions of skin friction and effectiveness for the present investigations is considered adequate for most design purposes. Predictions for effectiveness and skin friction are generally valid for the near-slot region $[x/S < 20, (x/S)\lambda^{-0.8} < 100]$ as well as far downstream.

The predicted local skin-friction distributions from Fig. 9 were integrated to determine the skin-friction drag reduction due to slot injection. The total drag reduction over a distance of 50 slot heights downstream is 40%–48% for all shots and both slot flow temperature ratios; the comparable reduction over a distance of 100 slot heights is 26–35%. These significant drag reductions would partially offset the drag penalties of a film-cooling system on a flight vehicle.

Results of an analytical exercise to determine the effect of slot flow temperature and heat conduction to the slot on effectiveness and skin friction for the 0.476-cm slot are shown in Fig. 10. Slot profiles were input so that the effect of conduction at a given $T_{t,j}/T_{t,\infty}$ could be observed (i.e., the measured profile from Fig. 3 is affected by heat conduction while a constant temperature profile equal to $T_{t,j}/T_{t,\infty}$ is free of conduction effects). Also, by comparing predictions for a constant temperature profile at each $T_{t,j}/T_{t,\infty}$ the effects of slot temperature level are evident. Heat conduction to the slot flow reduces effectiveness in the near-slot region $[(x/S)\lambda^{-0.8} < 100]$ and to a lesser degree farther downstream. Skin friction is not significantly affected by heat conduction for either temperature ratio. Decreasing the level of the slot flow temperature significantly decreases the slot effectiveness for $(x/S)\lambda^{-0.8} > 100$ but affects skin friction only in the near-slot region $(x/S < 10)$. In fact, skin friction for $x/S > 10$ is nearly independent of the present range of slot flow temperature.

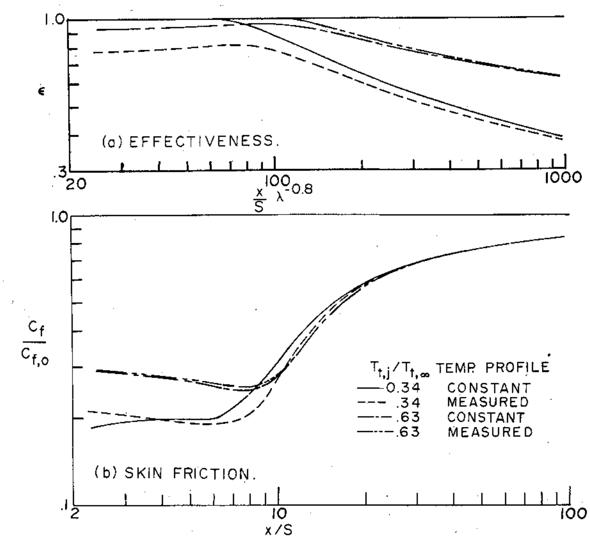

Fig. 10 Predicted effects of slot temperature and conduction on effectiveness and skin friction; $S = 0.475$ cm, $\lambda = 0.06$–0.07.

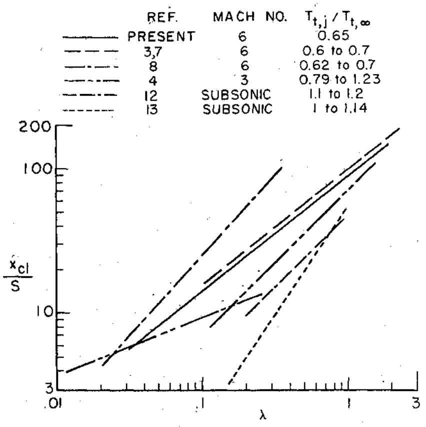

Fig. 11 Cooling length correlations.

Comparison with Previous results

The largest x/S value for which ε is greater than or equal to 1.0 is called cooling length, x_{cl}/S. Cooling length for the present investigation and $T_{t,j}/T_{t,\infty} = 0.63$ can be expressed as $x_{cl}/S = 89\lambda^{0.8}$. This expression was obtained from the previous power-law curve fit to the effectiveness data by setting $\varepsilon = 1$ and solving for x/S. The above equation is in good agreement with the cooling lengths found in Refs. 3 and 7 as shown in Fig. 11. Effectiveness results of Refs. 3, 7, and 8 are affected by the data reduction (see Ref. 2 for a discussion), but the experimental cooling lengths are insensitive to data reduction procedures and should be accurate. The cooling lengths for supersonic mainstream flow are larger without exception than for subsonic flow; also, cooling length distribution for supersonic ($M_j = 2.3$) slot injection from Ref. 8 (freestream Mach 6) is higher than any other results.

Comparison of the present effectiveness distribution with selected earlier results is shown for two values of $T_{t,j}/T_{t,\infty}$ in Fig. 12 for a small and larger value of λ. The subsonic results from Ref. 13 are representative of subsonic effectiveness data from Refs. 12–16 within $\pm 40\%$. Unpublished Mach 6 values of effectiveness obtained from direct measurements of surface equilibrium temperature at New York University for the same conditions and configuration as Refs. 3 and 7 were supplied by V. Zakkay and are included in Fig. 12. The present data derived from direct equilibrium surface temperature measurements indicate more film cooling effectiveness that the other Mach 6 results at comparable $T_{t,j}/T_{t,\infty}$. (However, significant differences in δ, t, and S exist between the present experiment and those of Refs. 3, 7, and 8.) All Mach 6 results indicate significantly greater effectiveness for film cooling in hypersonic flow than was found for subsonic or supersonic[4] flow.

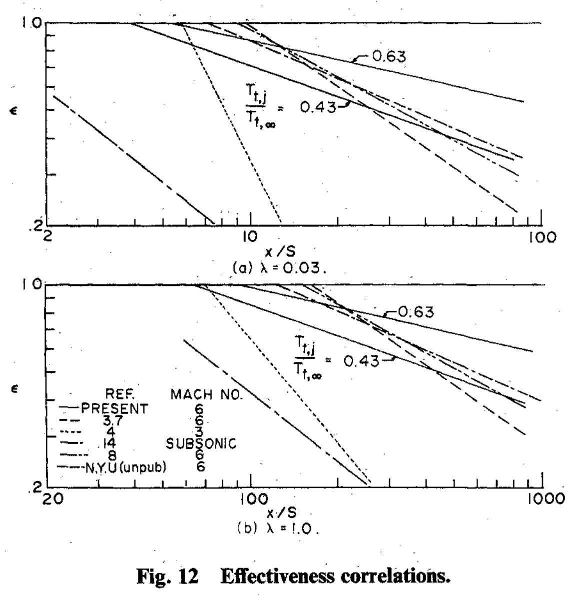

Fig. 12 Effectiveness correlations.

Conclusions

An experimental investigation of film cooling with sonic injection from a two-dimensional tangential slot into a thick turbulent boundary layer at Mach 6 resulted in the following conclusions:

1) Film cooling at Mach 6 is significantly more efficient than at lower speeds. The present direct surface temperature measurements indicate that film-cooling effectiveness deteriorates less rapidly with increasing distance from the slot than was indicated by previous data based on heat-transfer measurements at Mach 6.

2) Tangential slot injection reduced skin friction downstream of the slot, with the greatest reductions occuring near the slot.

3) Slot height in the range of this investigation did not significantly influence the film-cooling effectiveness correlation for constant lip thickness.

4) Film-cooling effectiveness was significantly reduced when the slot total-to-freestream total temperature ratio was decreased from 0.63 to 0.43.

5) Heat conduction from the mainstream through the slot lip significantly altered the slot temperature profile and thus modified the downstream effectiveness of the slot when the temperature of the injected gas was sufficiently different from that of the freestream flow.

6) A finite-difference method provided generally accurate predictions of effectiveness and skin friction when experimental profiles were used as initial inputs in the theory and the downstream static pressure was constant. Predictions were valid in the near-slot region ($x/S < 20$) as well as far downstream.

7) The improved efficiency of film cooling for high-speed flow over that for subsonic flow and the significant reductions in downstream drag obtained in the present investigation indicate that a new evaluation of film cooling as an active cooling system for use on hypersonic flight vehicles is required.

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Accurate Numerical Methods for Boundary-Layer Flows. II: Two-Dimensional Turbulent Flows

HERBERT B. KELLER*
California Institute of Technology, Pasadena, Calif.
AND
TUNCER CEBECI†
Douglas Aircraft Company, Long Beach, Calif.

A very simple and accurate numerical scheme which is applicable to quite general boundary-layer-flow problems has been devised. It has been tested extensively on laminar flows, turbulent flows (using eddy-diffusivity expressions), wake flows, and many other such flow problems. The procedure is much faster, easier to program and more flexible than most (if not all) other numerical methods which have been employed on such problems. This scheme also enables one to compute extremely close to the point of separation with no special precautions. The inclusion of chemically reacting species should offer no difficulties. The efficiency of the scheme clearly makes it applicable to three-dimensional steady, unsteady two-dimensional, and perhaps even unsteady three-dimensional, boundary-layer flows.

Nomenclature

A	= damping length, see Eq. (6)
c_f	= local skin-friction coefficient, $2\tau_w/\rho u_e^2$
f	= dimensionless stream function
H	= shape factor, δ^*/θ
p	= pressure
p^+	= pressure-gradient parameter
R_x	= local Reynolds number, $u_e x/\nu$
R_θ	= Reynolds number based on momentum thickness, $u_e\theta/\nu$
u_τ	= friction velocity, $(\tau_w/\rho)^{1/2}$
$\bar{u}, \bar{v}$	= x and y components of velocity
u_e	= edge velocity
v_w^+	= dimensionless velocity ratio, $\bar{v}_w/u_\tau$
x, y	= rectangular coordinates
β	= velocity-gradient parameter
δ	= boundary-layer thickness
δ^*	= displacement thickness, $\int_0^\infty (1 - \bar{u}/u_e)\, dy$
ε	= eddy viscosity
η	= transformed y-coordinate
θ	= momentum thickness, $\int_0^\infty \bar{u}/u_e (1 - \bar{u}/u_e) dy$
μ	= dynamic viscosity
ν	= kinematic viscosity
ξ	= transformed x-coordinate
ρ	= mass density
$-\rho\langle u'v'\rangle$	= Reynolds shear stress ‡
τ	= shear stress, wherever applicable
ψ	= stream function, wherever applicable

Subscripts

e	= outer edge of boundary layer
i	= inner region
o	= outer region
w	= wall

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* Professor of Applied Mathematics, Firestone Laboratories.

† Senior Engineer/Scientist, Aerodynamics Research Group. Member AIAA.

‡ Primes denote differentiation with respect to η.

Introduction

THERE is no shortage of numerical methods for solving, or more properly for approximating the solutions of various laminar and turbulent boundary-layer equations. The recent "Turbulent Olympics" volumes[1] give an excellent account of the current state-of-the-art for turbulent boundary-layer computations. The most crucial aspect of problems regarding turbulent flows is the basic formulation to be employed for the time-averaged fluctuating quantities that appear in the governing conservation equations. Since there is no generally accepted theory of turbulence, the field is essentially in a state of empiricism and many different theories or formulations yield results which agree well with a variety of experiments. On the other hand, each of these "theories" (i.e., eddy-viscosity, mixing length, turbulent energy, etc., Ref. 1) contain at least several parameters (if not functions) whose values are selected to best fit the results of some experiments. Thus, many of these theories resemble a sophisticated form of curve fitting. However, since they are of interest in many important problems in engineering, efficient and accurate numerical methods for their evaluation are of great importance. In addition, sufficiently flexible schemes enable three dimensional or time dependent problems to be treated and modifications to the empirical theories can easily be tested. This paper presents an extremely simple, accurate and efficient method that has been devised for these purposes.

The new method is much faster, easier to program and more flexible than most (if not all) other numerical methods which